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EDITED BY

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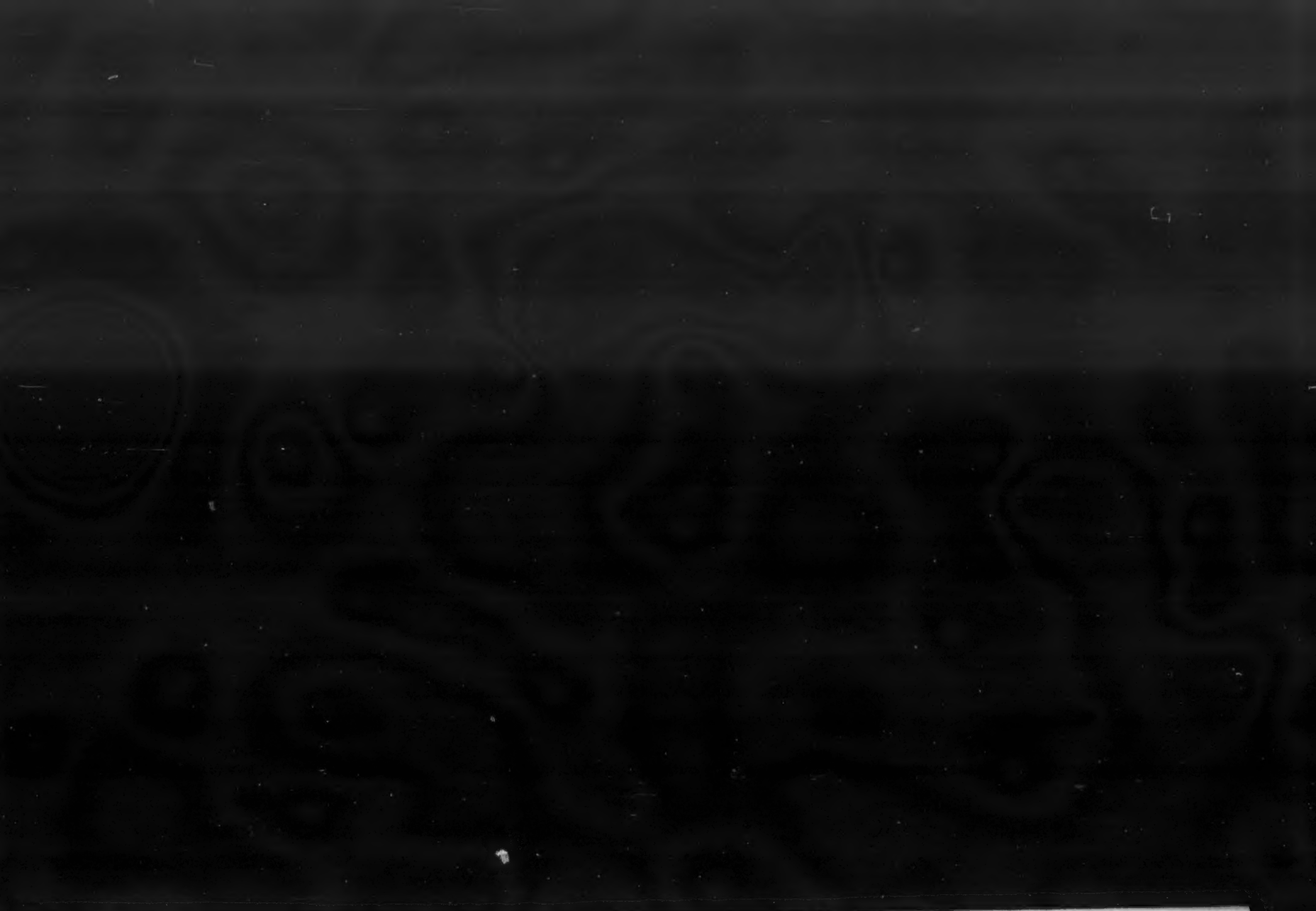

THE

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PRIZE PROBLEMS FOR STUDENTS.

I.

ANY side of a triangle is cut in the ratio of m to n, and the line joining this point to the opposite vertex is cut in the ratio of $m+n$ to l; to find the coördinates of the point of section.

II.

Can the equations $y^2+x=a$, $x^2+y=b$ be solved by quadratics?

III.

Find the polar equation of the line passing through the points of which the polar coördinates are r', φ'; r'', φ''.

IV.

Find the condition that $Ax+By+C=0$ should be tangent to $(x-a)^2+(y-b)^2=r^2$.

V.

Give the most concise method for finding the derivative of a^x.

The solutions of these problems must be received by the first of May, 1859.

MATHEMATICAL HOLOCRYPTIC CYPHERS.

By Pliny Earle Chase, Philadelphia.

The cyphers usually employed for secret correspondence, are formed by the mere substitution of symbols for the letters of the alphabet, one symbol only being appropriated to each letter. The comparative frequency with which the several letters recur in our language, is well known, therefore all cyphers of this kind are of little worth, as they may be easily translated by any expert into whose hands they may fall.

By the alternate use of two or three different symbols for each of the most common letters, the difficulty of translation is increased, and the value of the cypher proportionately enhanced. But even in this case the law is so simple, and the number of constants so great, that the key can generally be found after a few trials.

A holocryptic cypher should not only conceal the message that it is intended to convey, but it should also effectually hide the key to the message, which it can hardly do unless it destroys all symbolic uniformity, so that any given letter will be as likely to be represented by one symbol as by another. The various operations of Mathematics, — Addition, and the other simple rules of Arithmetic, Logarithms, Curvefunctions, &c., furnish an infinite variety of methods that will accomplish this end.

Let two correspondents, for instance, commence by arranging an alphabet in *rows* and *places*, in a manner similar to the following: —

		Places.									
		1	2	3	4	5	6	7	8	9	0
Rows.	1	X	U	A	C	O	N	Z	L	P	φ
	2	B	Y	F	M	&	E	G	J	Q	ω
	3	D	K	S	V	H	R	W	T	I	λ

According to this alphabet the word "Philip" may be designated

by $\frac{1\,3\,3\,1\,3\,1}{9\,5\,9\,8\,9\,9}$, the upper line denoting the *row*, and the lower line the *place*, of the several letters.

If we multiply the lower line by 9, we obtain for our cypher $\frac{1\,3\,3\,1\,3\,1}{8\,6\,3\,9\,0\,9\,1} = L\,n\,s\,i\,\varphi\,i\,x$.*

If the key that has been agreed upon, is the addition of 7 8 5 4 (repeated as often as necessary), 9 5 9 8 9 9 + 7 8 5 4 7 8 = 1 7 4 5 3 7 7. The left-hand figure of this sum may be rejected, as it will be readily known when the cypher is retranslated. We then obtain for our cypher $\frac{1\,3\,3\,1\,3\,1}{7\,4\,5\,3\,7\,7} = Z\,v\,h\,a\,w\,z$.

If a logarithmic key has been adopted, we may obtain the following cypher: log 9 5 9 8 9 9 † = 9 8 2 2 2 9; $\frac{1\,3\,3\,1\,3\,1}{9\,8\,2\,2\,2\,9} = P\,t\,k\,u\,k\,p$.

The following examples will illustrate the method of interpreting the three foregoing varieties of mathematical cypher.

		Key.
1.	$E\,\omega\,n\,t\,x\,b\,x\,x\,d\,h\,e\,o\,k\,j = \frac{2\,2\,1\,3\,1\,2\,1\,1\,3\,3\,2\,1\,3\,2}{6\,0\,6\,8\,1\,1\,1\,1\,1\,5\,6\,5\,2\,8}$	÷ 8
2.	$\Omega\,p\,c\,p\,w\,i\,\lambda\,m\,a\,o\,p\,i\,a\,g = \frac{2\,1\,1\,1\,3\,3\,3\,2\,1\,1\,1\,3\,1\,2}{0\,9\,4\,9\,7\,9\,0\,4\,3\,5\,9\,9\,3\,7}$	— 709
3.	$\Omega\,l\,g\,p\,n\,t\,u\,f\,k\,y\,g\,l = \frac{2\,1\,2\,1\,1\,3\,1\,2\,3\,2\,2\,1}{0\,8\,7\,9\,6\,8\,2\,3\,2\,2\,7\,8}$	log.

Dividing the place-row, in the first example, by 8, we obtain $\frac{2\,1\,3\,1\,2\,1\,1\,3\,3\,2\,1\,3\,2}{7\,5\,8\,5\,1\,3\,8\,8\,9\,4\,5\,6\,6}$ = Go to Baltimore.

Subtracting 7 0 9 7 0 9 7 0 9 7 0 9 7 0, from the place-row in the second example, we obtain $\frac{2\,1\,1\,1\,3\,3\,3\,2\,1\,1\,1\,3\,1\,2}{3\,8\,5\,2\,6\,9\,3\,3\,3\,8\,8\,9\,6\,7}$ = Flour is falling.

In the third example, 0 8 7 9 6 = log 1 2 2 4 5; 8 2 3 2 2 = log 6 6 5 6 2; log 7 8 = 6. Then $\frac{2\,1\,2\,1\,1\,3\,1\,2\,3\,2\,2}{1\,2\,2\,4\,5\,6\,6\,5\,6\,2\,6}$ = Buy corn and rye.

These illustrations, I think, are sufficient to show that a very simple arithmetical process may effectually conceal the meaning of a message from every one but the persons who hold the key to the cypher. The obscurity and consequent security may be increased

---

* The first letter may be either L, J, or T.

† The index of the logarithm, is disregarded.

to any desired extent, by adopting processes that are more intricate. But the simpler the cypher, provided it is effectual, the better, and I know of no simpler methods, that merit the name of holocryptic, than those I have just indicated.

MOMENT OF INERTIA.

BY GERARDUS B. DOCHARTY, LL. D.
Professor of Mathematics in the Free Academy, New York City.

WHEN we arrive at the conclusion, that the *moment of Inertia* is the sum of all the elements of revolving matter into the square of their distance from the axis of revolution, all the after difficulties resolve themselves into analytical ones; and these can only refer to the limits of integration.

The following article is offered, not as having any merit beyond what may be found in the works of Prof. BARTLETT and others who have discussed this subject; but simply for the purpose of assisting those to interpret their symbols, who are pursuing the study of mathematics without the aid of a teacher.

1st. *For the cylinder round an axis perpendicular to its own.*

Let $2a =$ its length, $r =$ radius of its base, then

$$k^2 M = \int r^2 \, dM = \iiint (x^2 + z^2)\, dx\, dy\, dz$$

$$= \int_{-r}^{r} dz \int_{-\sqrt{r^2-z^2}}^{\sqrt{r^2-z^2}} dy \int_{-a}^{a} dx\, (x^2 + z^2)$$

$$(1) \qquad = 2a \int_{-r}^{r} dz \int_{-\sqrt{r^2-z^2}}^{\sqrt{r^2-z^2}} dy\, (\tfrac{1}{3} a^2 + z^2)$$

$$(2) \qquad = 4a \int_{-r}^{r} dz\, (\tfrac{1}{3} a^2 + z^2) \sqrt{r^2 - z^2}$$

Let $z = r \sin \varphi$. Then

$$k^2 M = 2a r^2 \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} d\varphi (\tfrac{1}{3} a^2 + \tfrac{1}{4} r^2 + \tfrac{1}{3} a^2 \cos 2\varphi - \tfrac{1}{4} r^2 \cos 4\varphi)$$

$$= 2a r^2 \pi (\tfrac{1}{3} a^2 + \tfrac{1}{4} r^2) = M (\tfrac{1}{3} a^2 + \tfrac{1}{4} r^2)$$

$$\therefore k^2 = \tfrac{1}{3} a^2 + \tfrac{1}{4} r^2$$

The first integral, $2a(\frac{1}{3}a^2 + z^2)$, is the moment of any elementary line of matter within the cylinder in the direction of the axis of the cylinder.

The second integral, $4a(\frac{1}{3}a^2 + z^2)\sqrt{r^2 - z^2}$, is the moment of any elementary plane within the cylinder, parallel to the plane of xy. That is, it is the sum of the moments of all the previous elementary lines which have the same z.

If we had put $y = r\sin\varphi$, $z = r\cos\varphi$, then $dM = r\,dr\,dx\,d\varphi$. This elementary volume is not the same as the previous one.

$$\begin{aligned} k^2 M &= \iiint (x^2 + r^2\cos^2\varphi)\, r\,dr\,dx\,d\varphi \\ &= \int_0^{2\pi} d\varphi \int_0^r r\,dr \int_{-a}^{a} (x^2 + r^2\cos^2\varphi)\,dx \\ &= 2a\int_0^{2\pi} d\varphi \int_0^r r\,dr\,(\tfrac{1}{3}a^2 + r^2\cos^2\varphi) \\ &= a r^2 \int_0^{2\pi} d\varphi\,(\tfrac{1}{3}a^2 + \tfrac{1}{4}r^2 + \tfrac{1}{4}r^2\cos 2\varphi) \\ &= 2a r^2\pi\,(\tfrac{1}{3}a^2 + \tfrac{1}{4}r^2) = M(\tfrac{1}{3}a^2 + \tfrac{1}{4}r^2) \end{aligned}$$

$$\therefore k^2 = \tfrac{1}{3}a^2 + \tfrac{1}{4}r^2 \text{ as before.}$$

The first integral is the same as the first above. The second, $a r^2(\frac{1}{3}a^2 + \frac{1}{4}r^2 + \frac{1}{4}r^2\cos 2\varphi)$, is the moment of an elementary plane drawn from the axis of the cylinder to the circumference. The variation of φ from 0 to 2π in the next integral produces the sum of the moments of the whole mass.

The integrations are the same for all homogeneous bodies, differing only in the limits of integration.

2d. *If the bar had been a square prism,* instead of a cylinder, *about an axis parallel to one of the lateral faces and equidistant from them.*

Let $2b$ = the length of one side of the base. Then

$$\begin{aligned} k^2 M &= \int r^2\,dM = \iiint (x^2 + z^2)\,dx\,dy\,dz \\ &= \int_{-b}^{b} dz \int_{-b}^{b} dy \int_{-a}^{a} dx\,(x^2 + z^2) = 2a\int_{-b}^{b} dz \int_{-b}^{b} dy\,(\tfrac{1}{3}a^2 + z^2) \\ &= 4ab\int_{-b}^{b} dz\,(\tfrac{1}{3}a^2 + z^2) = \tfrac{8}{3}ab^2(a^2 + b^2) = \tfrac{1}{3}M(a^2 + b^2) \end{aligned}$$

$$\therefore k^2 = \tfrac{1}{3}(a^2 + b^2).$$

3d. *But, if the axis of motion pass diagonally through the mid-section,* it will be

$$k^2 M = \int_{-b\sqrt{2}}^{b\sqrt{2}} dz \int_{-b\sqrt{2}+z}^{b\sqrt{2}-z} dy \int_{-a}^{a} dx\,(x^2+z^2)$$
$$= 2a \int_{-b\sqrt{2}}^{b\sqrt{2}} dz \int_{-b\sqrt{2}+z}^{b\sqrt{2}-z} dy\,(\tfrac{1}{3}a^2+z^2)$$
$$= 4a \int_{-b\sqrt{2}}^{b\sqrt{2}} dz\,(b\sqrt{2}-z)\,(\tfrac{1}{3}a^2+z^2)$$
$$= 4a \int_{-b\sqrt{2}}^{b\sqrt{2}} dz\,(\tfrac{1}{3}a^2 b\sqrt{2} - \tfrac{1}{3}a^2 z + b\sqrt{2}.z^2 - z^3)$$
$$= \tfrac{16}{3} a b^2 (a^2+2b^2) = \tfrac{8}{3} M (a^2+2b^2)$$
$$\therefore k^2 = \tfrac{8}{3}(a^2+2b^2).$$

The final result *must be the same,* whatever be the *order* of the integrations; but the *difficulty* will not be the same, in general, nor will the meaning of the operations.

For example, in the second set of integrations, if we order them thus: —

$$k^2 M = \int_{-a}^{a} dx \int_0^{2\pi} d\varphi \int_0^r r\,dr\,(x^2 + r^2\cos^2\varphi)$$
$$= \tfrac{1}{4} r^2 \int_{-a}^{a} dx \int_0^{2\pi} d\varphi\,(2x^2 + r^2\cos^2\varphi) = \tfrac{1}{4} r^2 \pi \int_{-a}^{a} dx\,(4x^2+r^2)$$
$$= \tfrac{1}{2} r^2 a\pi\,(\tfrac{4}{3}a^2 + r^2) = M(\tfrac{1}{3}a^2 + \tfrac{1}{4}r^2) \therefore k^2 = \tfrac{1}{3}a^2 + \tfrac{1}{4}r^2.$$

The first integral, $\frac{1}{4} r^2 (2x^2 + r^2\cos^2\varphi)$, is the moment of a line, drawn perpendicular from the axis of the cylinder to the circumference.

The second integral, $\frac{1}{4} r^2 \pi (4x^2 + r^2)$, is the moment of any circular section perpendicular to the axis of the cylinder.

4th. *For the cylinder revolving on its own axis.*

$$k^2 M = \iiint (y^2+z^2)\,dx\,dy\,dz = \iiint r^2\,dx\,r\,dr\,d\varphi$$
$$= \int_0^r r^3\,dr \int_0^{2\pi} d\varphi \int_{-a}^{a} dx = 2a \int_0^r r^3\,dr \int_0^{2\pi} d\varphi$$
$$= 4a\pi \int_0^r r^3\,dr = a r^4 \pi = \tfrac{1}{2} r^2 M \therefore k^2 = \tfrac{1}{2} r^2.$$

5th. *For the cone revolving on its own axis.*

$$k^2 M = \iiint (y^2 + z^2)\, dx\, dy\, dz = \iiint r^2\, dx\, r\, dr\, d\varphi$$
$$= \int_0^{b\cos\vartheta} dx \int_0^{2\pi} d\varphi \int_0^{x\tan\vartheta} r^3\, dr = \tfrac{1}{4}\tan^4\vartheta \int_0^{b\cos\vartheta} x^4\, dx \int_0^{2\pi} d\varphi$$
$$= \tfrac{1}{2}\pi\tan^4\vartheta \int_0^{b\cos\vartheta} x^4\, dx = \tfrac{1}{10}\pi\tan^4\vartheta\, b^5\cos^5\vartheta$$
$$= \tfrac{1}{10}\pi\, b^5 \sin^4\vartheta\cos\vartheta = \tfrac{3}{10} b^2\sin^2\vartheta\,.\,M$$
$$\therefore k^2 = \tfrac{3}{10} b^2\sin^2\vartheta = \tfrac{3}{10} r^2.$$

Where b is the slant height of the cone; ϑ the angle which b makes with the axis; r the radius of the base of the cone, and k the altitude.

The first integral gives the moment of a radius drawn perpendicular from the axis to the surface of the cone, at the distance x from the vertex.

The second, that of a circular section at that distance. If we say

$$k^2 M = \iiint r^3\, dx\, dr\, d\varphi = \int_0^r r^3\, dr \int_0^{2\pi} d\varphi \int_{r\cot\vartheta}^{b\cos\vartheta} dx$$
$$= \int_0^{b\sin\vartheta} r^3\, dr \int_0^{2\pi} d\varphi\, (b\cos\vartheta - r\cot\vartheta)$$
$$= 2\pi \int_0^{b\sin\vartheta} r^3\, dr\, (b\cos\vartheta - r\cot\vartheta)$$
$$= 2\pi\, (\tfrac{1}{4}\, b^5\sin^4\vartheta\cos\vartheta - \tfrac{1}{5}\, b\sin^5\vartheta\cot\vartheta)$$
$$= \tfrac{1}{10}\pi\, b^5\sin^4\vartheta\cos\vartheta = \tfrac{3}{10} b^2\sin^2\vartheta\,.\,M$$
$$\therefore k^2 = \tfrac{3}{10} b^2\sin^2\vartheta = \tfrac{3}{10} r^2 \text{ as before.}$$

The first integral gives the moment of a line (of elementary particles) parallel to the axis of x, embraced between the curvilinear surface of the cone and its base.

The second, the moment of all such lines which have the same r, or that of a cylindrical surface of elementary particles at any distance r from the axis.

The third, the sum of the moments of *all* such, or that of the whole cone.

If the student shall carefully make himself master of the foregoing examples, he will find no difficulty in determining the moment of inertia of any other homogeneous body of regular figure.

THE THEOREM OF PAPPUS.

By J. B. Henck, Civil Engineer, Boston.

The famous theorem for measuring surfaces and solids of revolution, so often ascribed to Guldin, may be expressed in two parts thus:

The surface of revolution described by a line lying wholly on one side of the axis, and in the same plane with it, is measured by the product of the revolving line into the length of the path described by its centre of gravity.

The solid of revolution described by a surface lying wholly on one side of the axis, and in the same plane with it, is measured by the product of the revolving surface into the length of the path described by its centre of gravity.

The claims of Guldin to this theorem have been repeatedly discussed, and the fact made manifest, that the theorem was in existence more than 1200 years before the time of Guldin. Pappus of Alexandria had anticipated the learned Jesuit. Yet many authors, in treating of this theorem, do not even mention Pappus, but either simply call it the "Theorem of Guldin," or expressly ascribe the authorship to him. It may not, therefore, be amiss to present the case more fully, with the citation in the original, of such documents as are of importance, so that mathematicians may decide, whether, in justice to one of the four great geometers of antiquity, the theorem should not be referred to invariably as the *Theorem of* Pappus.

Pappus of Alexandria flourished, according to the common account, about A. D. 380. His principal work is his "Mathematical

Collections." In the Preface to the Seventh Book, after giving an account of the works of the ancient geometers, and of some problems that engaged their attention without being solved, he brings forward the theorem in question, as a proof that his own efforts to contribute something useful have not been in vain. The theorem itself, and a remark appended, are given in these words:*

Ὁ μὲν τῶν τελείων ἀμφοιστικῶν λόγος συνῆπται ἔκτε τῶν ἀμφοισμάτων καὶ τῶν ἐπὶ τοὺς ἄξονας ὁμοίως κατηγμένων εὐθειῶν ἀπὸ τῶν ἐν αὐτοῖς κεντροβαρικῶν σημείων. Ὁ δὲ τῶν ἀτελῶν ἔκτε τῶν ἀμφοισμάτων καὶ τῶν περιφερειῶν ὅσας ἐποίησε τὰ ἐν αὐτοῖς κεντροβαρικὰ σημεῖα· ὁ δὲ τούτων περιφερειῶν δῆλον ὡς ἔκτε τῶν κατηγμένων καὶ ὧν περιέχουσιν αἱ τούτων ἄκραι, εἰ καὶ εἶεν πρὸς τοῖς ἄξοσιν ἀμφοιστικῶν, γωνιῶν.

Περιέχουσι δὲ αὗται αἱ προτάσεις, σχεδὸν οὖσαι μία, πλεῖστα ὅσα καὶ παντοῖα θεωρήματα γραμμῶν τε καὶ ἐπιφανειῶν καὶ στερεῶν, πάνθ' ἅμα καὶ μιᾷ δείξει· καὶ τὰ μὴ προδεδειγμένα καὶ τὰ ἤδη, ὡς καὶ τὰ ἐν τῷ δωδεκάτῳ τῶν Στοιχείων.

These passages may be translated as follows:

"The ratio of magnitudes described by a complete revolution† is compounded of the ratio of the revolving figures, and of the ratio of the straight lines similarly drawn from their centres of gravity to the axes. The ratio of magnitudes described by an incomplete revolution is compounded of the ratio

---

* From the original of this preface given by HALLEY in his edition of Apollonius de Sectione Rationis. Oxon. 1706. The Greek text of PAPPUS, with the exception of some detached portions, such as that just cited, has never been printed. The world has long expected from the University of Oxford an edition of PAPPUS similar to its editions of Euclid, Apollonius, and Archimedes.

† The words *ἀμφοιστικῶν* and *ἀμφοισμάτων* of the original are unusual forms, and are not given in the lexicons to which I have access. They are doubtless derived from *ἀμφί* and the root *ΟΙΩ*, *οἴσω*, which supplies several of the forms of *φέρω*; and the meaning of the first appears to be "the effect produced by carrying something around," that is, a solid or surface of revolution, and the meaning of the second, "the thing carried around," that is, the generating area or line. The words *εἰ καὶ εἶεν* present some difficulty. They seem to imply that the angles spoken of might be somewhere else than at the axes, which does not appear possible. Still the phrase may be equivalent to "*which are*," and this is the view taken in our translation.

of the revolving figures, and of the ratio of the arcs described by their centres of gravity; and the ratio of these arcs is evidently compounded of the ratio of the lines drawn to the axes, and of the ratio of the angles contained by the extremities of these lines at the axes of the magnitudes described."

He adds: "These propositions, which are almost the same, embrace many and various theorems of lines, surfaces, and solids, all with one demonstration. Of these theorems some have not been before demonstrated, while others have been, such as those in the twelfth book of the Elements."

This enunciation of the theorem differs from the modern one only in this, that, instead of giving directly the measure of a solid or surface of revolution, PAPPUS gives it by giving the *ratio* to each other of any two of these solids or surfaces. From the remark added, he appears to have applied the theorem in various ways, including the measurement of the cone, cylinder, and sphere, which are the solids of revolution of the twelfth book of the Elements of Euclid, to which he doubtless refers.

We of course lose sight of this theorem in the darkness that soon obscured all science. A new day came at last, brightened almost at once by the genius of VIETA, KEPLER, GALILEO, DES CARTES, and CAVALIERI. Not far from this time COMMANDINUS produced his translations and commentaries, and gave a new impulse to the study of the ancient geometers. Among these versions was that of PAPPUS, published in 1588,* thirteen years after the death of COMMANDINUS. His translation of our theorem, as it stands in the reprint of 1660, is thus expressed:

Perfectorum utrorumque ordinum proportio composita est ex proportione amphismatum, et rectarum linearum similiter ad axes ductarum a punctis,

---

* Pappi Alexandrini Mathematicæ Collectiones a Federico Commandino in Latinum conversæ et Commentariis illustratæ. Fol. Pisauri, 1588; item Bononiæ, 1660.

*quæ in ipsis gravitatis centra sunt. Imperfectorum autem proportio composita est ex proportione amphismatum, et circumferentiarum a punctis, quæ in ipsis sunt centra gravitatis, factarum. Harum circumferentiarum proportio dividitur in proportionem ductarum linearum, et earum, quas continent ipsarum extrema ad axes angulorum.**

In 1615 KEPLER published his Nova Stereometria.† In this he introduces the method of generating solids by the rotation of figures about an axis. Of these solids he describes a great number, and gives them various fanciful names, as the olive, apple, citron, &c. Among these he has circular and elliptic rings, the measure of which he obtains by multiplying the area of the rotating circle or ellipse by the length of the circumference described by their centres. He does not, however, use the centre of gravity, as such, in any of his measures.

We come now to GULDIN, who was born in 1577, at St. Gall in Switzerland, and named Habakkuk. At the age of twenty he renounced Protestantism, and entered the Society of Jesuits as a "temporal coadjutor," changing his name to Paul. He was sent to Rome by his superiors to study mathematics; and he afterwards taught that science in the colleges of his order at Rome, Grätz, and Vienna. His principal work is his Treatise, in four books, on the Centre of Gravity.‡ He died in 1643.

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* In this version the words "*utrorumque ordinum*" answer to ἀμφοτικῶν, where it first occurs in the Greek text of HALLEY, and a blank supplies its place near the end; while ἀμφισμάτων is simply changed to *amphismatum*. COMMANDINUS certainly enlightens us very little as to the meaning of these words. Towards the close, "*earum quas*" seems to be a misprint for *eorum quos*, referring to *angulorum*.

† Nova Stereometria Doliorum vinariorum, etc. Accessit Stereometriæ Archimedeæ Supplementum. Lincii, 1615.

‡ Pauli Guldini Sancto-Gallensis, e Societate Jesu, de Centro Gravitatis trium specierum quantitatis continuæ. Liber Primus. Fol. Viennæ Austriæ, 1635; Liber Secundus, 1640; Liber Tertius, 1641; Liber Quartus, 1641.

GULDIN appears to have been a close student of PAPPUS, whom he cites, in the course of his book, about a dozen times. In several instances he quotes sentences, and these he gives in the very words of the version of COMMANDINUS. In his second book, after considerable heralding, he introduces as his own discovery the following Rule:

Quantitas rotanda in viam rotationis ducta, producit potestatem rotundam uno gradu altiorem, potestate sive quantitate rotata.

"*The quantity to be rotated, multiplied by the path of rotation, produces a circular power one degree higher than the power or quantity rotated.*"

In explanation of this rule GULDIN observes, that there are three geometrical powers; namely, the line, a power of the first degree, produced by the motion of a point; the surface, a power of the second degree, produced by the motion of a line; and the solid, a power of the third degree, produced by the motion of a surface. These powers are "circular powers," when the generating motion is circular. The "path of rotation" is the path described by the centre of gravity. His third Corollary is:

Si autem tam quantitates rotandæ, quam viæ sive radii rotationis sint inæquales, sequitur ulterius potestatum proportionem esse compositam ex ratione quantitatis rotatæ unius, ad quantitatem rotatam alterius; et ex ratione viæ vel radii illius unius, ad viam vel radium hujus alterius.

"*But if both the quantities to be rotated and the paths or radii of rotation are unequal, it follows, further, that the ratio of the powers is compounded of the ratio of the rotated quantity of the one to the rotated quantity of the other, and of the ratio of the path or radius of the former to the path or radius of the latter.*"

This is of course a reproduction of the theorem of PAPPUS, and the third Corollary presents it in almost the same form. It would not, perhaps, be just to accuse GULDIN of deliberate plagiarism; for,

though he probably read the passage cited above from the version of COMMANDINUS, he might possibly fail to get a correct idea of the theorem from this faulty translation. Still the ingenuity to do so would seem to be much less than that required to enable him to discover it independently. His attempts at originality on other subjects are not so successful as to give us a very high opinion of his inventive powers; while the general tone of his work, his evident conceit, self-satisfaction, and boastfulness, do not predispose the reader to admit his claims in the present instance without some scrutiny.

In regard to a demonstration of the theorem, we do not know that PAPPUS ever gave one. A preface was not the place for a demonstration, and if he resumed the subject elsewhere, we do not know it. GULDIN, however, seems to have attempted a demonstration, which amounts to this: The quantity generated is equal to the revolving figure multiplied by the length of some one of the circumferences described by its points. The circumferences described by the points nearest the axis are too small for this purpose, and those described by the points farthest from the axis are too large. Therefore, a circumference must be found at such a distance from the axis that the excess outside of this distance just balances the deficiency within it, and this line must be one that is unique (*unica*). All this will be done, if we take for that circumference the path described by the centre of gravity, which is a unique line (*quæ est sola et unica*). He does not, however, show how the balancing of which he speaks takes place, which is the essential point, but contents himself with asserting that such is the fact. He adds, that this thing requires no other demonstration; but it will be sufficient to show its truth by induction; that is, by ascertaining that the results by his rule agree precisely, in numerous examples, with those obtained otherwise.

The first real demonstration appears to have been given in a

work by CAVALIERI.* GULDIN, in his fourth book, attacks the Method of Indivisibles of CAVALIERI, accusing him of departing from the rigor of the ancient geometry. CAVALIERI, according to MONTUCLA, retorts, that GULDIN's demonstration, just alluded to, displays any thing but the ancient rigor, and says, that one of his pupils, ANTONIO ROCCHA, furnished him with a demonstration of the theorem, long before GULDIN published his work. This demonstration he gives.

It may be interesting, before closing, to give briefly some of the opinions of writers who have touched upon the claims of PAPPUS and GULDIN.

LEIBNITZ,† speaking of this theorem, and giving an extension of it, remarks: "PAPPUS had briefly indicated what GULDIN showed more fully." (*Pappus subindicaverat quod Guldinus expressius ostendit.*)

HALLEY ‡ says, that one of his principal motives in giving the Greek text of the preface to the Seventh Book of PAPPUS was, "that it might become plain to all, that the Centrobaric Method of GULDIN was known to the ancients also." (*Ut palam fiat omnibus Regulam Guldini Centrobaricam . . . . . ipsis etiam Veteribus innotuisse.*)

VARIGNON,§ in a memoir in which he demonstrates the theorem itself and several extensions of it, does not even allude to PAPPUS, but ascribes the discovery to GULDIN.

MONTUCLA ‖ discusses the whole question fully, and then asks: "After these developments, ought we not to give to this fertile prin-

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* Exercitationes Geometricæ Sex. Bononiæ, 1647. This work, and several others of the same age, which would more fully illustrate the views of Guldin's immediate successors, are unfortunately not to be found in the libraries of this vicinity.

† Acta Eruditorum, 1695, p. 493.

‡ Preface to Apollonius de Sectione Rationis. Oxon. 1706.

§ Mémoires de l'Académie. 1714, p. 77.

‖ Histoire des Mathématiques, Vol. I. p. 329.

ciple the name of PAPPUS, instead of that of GULDIN, who merely reproduced it, whether he did it by his own ingenuity, or whether this passage of PAPPUS gave him the idea?" He adds, after mentioning GULDIN's evident acquaintance with PAPPUS: "Still I do not care to accuse GULDIN of plagiarism; although it seems to me difficult to free him from the charge."

BOSSUT,* speaking of PAPPUS, says: "I ought to add, in praise of PAPPUS, that we find at the end of the Preface to his Seventh Book a quite distinct idea (*une idée assez distincte*) of the famous theorem commonly attributed to the Jesuit GULDIN."

FRISI,† after demonstrating the theorem, remarks: "This is the celebrated theorem which some contend was indicated by PAPPUS ALEXANDRINUS, but which was really first given for circular and elliptic rings by KEPLER, and afterwards proposed generally by GULDIN, and proved by a sort of induction, and which, finally, was demonstrated by ANTONIO ROCCA of Piacenza, by means of the principles of our CAVALIERI, in nearly the same way that we have here followed."

WHEWELL ‡ says: "A property of the center of gravity, discovered by PAPPUS, and republished about 1640 by GULDIN, has since borne the name of its more recent enunciator."

And, lastly, CHASLES § says: "The famous theorem of GULDIN . . . . . is found in the Mathematical Collections, and appears to have been the discovery of PAPPUS himself." And afterwards, speaking of GULDIN, he adds: "This rule was unnoticed, when GULDIN dis-

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* Histoire des Mathématiques, Vol. I. p. 47.

† Paulli Frisii Opera, 3 Vols. 4to. Mediolani, 1782–85, Vol. II. p. 30.

‡ Mechanics. Cambridge, 1819, p. 111.

§ Aperçu Historique sur l'Origine et le Développement des Méthodes en Géométrie, Bruxelles, 1837, p. 29 et 57.

covered it in his turn, and used it to solve problems that were difficult and intractable by other methods."

All these writers recognize the claims of PAPPUS, except VARIGNON, who does not mention them, and FRISI, who seems inclined to ignore them. Among writers not yet cited, PRONY, BORGNIS, BOUCHARLAT, YOUNG, MOSELEY, DUHAMEL, and WEISBACH do not mention PAPPUS in connection with this theorem, but refer to it as GULDIN'S Theorem; FRANCŒUR and O. GREGORY ascribe its discovery to PAPPUS; while WALTON * and PRICE † refer to it as the Theorem of PAPPUS.

The evidence, so far as we have been able to collect it, is now before the reader. Let him judge for himself. It is a beautiful custom, nowhere more fully recognized than among mathematicians, to connect with a new truth the name of its author; and if in this case the wrong name has received the honor, it is not too late to correct the error. We have only to reverse the prevalent practice; so that, instead of mentioning PAPPUS incidentally or not at all, and quoting the theorem as GULDIN'S, we shall, if occasion serve, mention the *applications* made by GULDIN, but the theorem itself we shall quote as the THEOREM OF PAPPUS.

ON THE COURSING JOINT CURVE OF AN OBLIQUE ARCH IN THE FRENCH SYSTEM.

BY DEVOLSON WOOD, C. E.
Assistant Prof. of Engineering in the Univ. of Michigan.

1. IN constructing the development of the arch the draughtsman must either construct the coursing joints geometrically or else

* Problems in Theoretical Mechanics.

† Infinitesimal Calculus, 3 Vols. 8vo. 1852–1856.

construct them from the equation of the curve. I propose to find the equation of the curve when the face is semicircular.

Suppose the arch is intersected by an infinite number of planes parallel to the face, they will cut an infinite number of ring joints. In the French system the coursing joints of either the intrados or extrados intersect these joints at right angles.

Let B' be the origin, and $B'E$ be the axis of x which is the springing line; $r = B'C =$ radius; $r\varphi =$ the length of arc from the springing line to the curve $= a\,d$; $r\varphi_1 =$ initial arc $= B'D'$; $i =$ inclination of the curve to an element; $\theta = A\,C\,B' - 90° =$ skew angle.

2. To find i in terms of φ and θ. With C as a centre and radius equal to unity describe the arcs $A\,B$, $A\,D$, and $D\,B$. We shall thus form a spherical triangle $A\,D\,B$, in which $A\,B = 90° + \theta$, $D\,B = \varphi$, $A\,D\,B = 90° + i$, $A\,B\,D = 90°$. Hence, by NAPIER'S rules,

$$\sin\varphi = \cot(90° + i)\tan(90° + \theta),$$

or

$$\sin\varphi = \tan i \cot\theta,$$

$$\therefore\ \tan i = \sin\varphi\tan\theta.$$

This value of i is the same for any value of the radius. In the differential triangle $a\,b\,c$ we have

$$\frac{b\,c}{a\,b} = \frac{r\,d\,\varphi}{d\,x} = \tan i = \sin\varphi\tan\theta.$$

Hence

$$\int_0^x d\,x = r\cot\theta\int_{\varphi_1}^{\varphi}\frac{d\,\varphi}{\sin\varphi};$$

or

$$x = r\cot\theta\,(\log\tan\tfrac{1}{2}\varphi - \log\tan\tfrac{1}{2}\varphi_1),$$

from which the curve in the development may be constructed.

In a German work on bridges, the following formula is given, but without investigation; namely,

$$x = \frac{r}{\sin\theta} \log \cot \tfrac{1}{2}\varphi,$$

which is evidently incomplete, since φ can have but one value for any assumed value of x; whereas it should have several values, one for each coursing joint which depends upon the initial value φ_1 of φ for $x = 0$.

THE MOTIONS OF FLUIDS AND SOLIDS RELATIVE TO THE EARTH'S SURFACE.

[Continued from page 148.]

SECTION II.

ON THE MOTIONS AND FIGURE OF A FLUID SURROUNDING THE EARTH.

11. The results obtained in this and the following section, will be upon the hypothesis that the motions of the fluid are not resisted by the earth's surface. The motions and figure of the fluid must be such as to satisfy equations (13), and also the condition of continuity. To determine them in the general case in which both α and g are functions of h, θ' and φ, would be very difficult. We shall take here the special case only in which α is a function of θ' which increases or decreases, from the equator to the pole, and in which g is regarded as constant at all parts of the earth's surface, and throughout the whole range of altitude. Equations (13) become in this case, when the fluid is elastic,

$$(20)\quad \begin{aligned} g\,D_\theta h &= r^2 \sin\theta\cos\theta\,(2n + D_t\varphi)\,D_t\varphi - r^2 D_t^2\theta - g h D_\theta \log\alpha \\ g\,D_\phi h &= -2r^2\sin\theta\cos\theta\,(n + D_t\varphi)\,D_t\theta - r^2\sin^2\theta\,D_t^2\varphi. \end{aligned}$$

In inelastic fluids we have $\log k$ instead of $\log a$.

12. To determine completely the motions and figure of the fluid which, would satisfy the conditions for any initial state of the fluid, upon the hypothesis that the motions are entirely free from resistances arising from the motions of the particles amongst themselves, would be impossible. But since in all fluids there are slight resistances to the motions of the particles amongst themselves, which, however small, eventually destroy all oscillatory or wave motions depending upon the initial state of the fluid, and reduce the motions of the particles amongst themselves to the minimum which satisfies the conditions, it is not necessary to integrate the equations generally, but merely to satisfy them with the least possible motion of the particles amongst themselves. Hence both the motions and density of the fluid at any place, and likewise its figure, must be independent of the time, and therefore constant.

13. Since $D_\varphi h$ in the last of equations (20) can only have a value arising from an oscillatory or wave motion of the fluid, which would soon be destroyed, it must be put equal 0, and then the equation gives by integration for each particle supposed to be entirely free from the resistances arising from the motions of the particles amongst themselves,

$$r^2 \sin^2 \vartheta \, (n + D_t \varphi) = c, \tag{21}$$

in which c is a constant depending upon the initial motion of the particle. Let

R be the radius of the earth regarded as constant,

m the mass of the fluid, and

l its uniform depth when at rest relative to the earth.

As the quantity of motion in the whole mass cannot be affected by the mutual actions of the particles upon each other, we have, even in the case in which the particles are not free from mutual resistances,

$$\int_m r^2 \sin^2 \vartheta \, (n + D_t \varphi) = \int_m c = Cm, \tag{22}$$

in which C is a constant depending upon the initial motions of all the particles.

The first member of this equation expresses the sum of the areas projected upon the plane of the equator, arising from the absolute motions of all the particles for a unit of time, and hence this sum is constant.

14. If we put v for $D_t \varphi$ belonging to the initial state of the fluid, the last equation gives, neglecting quantities of the order of the range of altitude compared with the earth's radius,

$$\begin{aligned} Cm &= R^2 \int_m \sin^2 \vartheta \, (n + v), \\ &= R^4 \int_0^l \int_N^{2\pi} \int_\phi^\pi \int_0^\theta k \sin^3 \vartheta \, (n + v), \\ &= \tfrac{2}{3} R^2 m \, (n + v') \end{aligned}$$

in which

$$v' = \frac{3R^2}{2m} \int_0^l \int_N^{2\pi} \int_\phi^\pi \int_0^\theta k \, v \sin^3 \vartheta.$$

Hence,

$$C = \tfrac{2}{3} R^2 \, (n + v').$$

In the preceding integration k is supposed to be independent of ϑ. If the density should vary considerably with the latitude, it would affect the preceding result slightly.

When, by the mutual actions of the different strata upon one another, $D_t \varphi$ becomes the same at all altitudes, upon the same parallel of latitude, c then becomes equal to C, and equation (21) gives

$$D_t \varphi = \frac{C}{R^2 \sin^2 \theta} - n = \frac{2 \, (n + v')}{3 \sin^2 \theta} - n. \tag{23}$$

This value of $D_t \varphi$ satisfies the last of equations (20), and since it gives a uniform motion of all the particles of the fluid upon the same parallels of latitude, as much fluid flows from any place as flows into it, while the density, from what has been stated, remains the same, and hence it also satisfies the condition of continuity.

15. If we suppose the initial state of the fluid to be that of rest relative to the earth's surface, equation (23) becomes

$$D_t \varphi = \left(\frac{2}{3 \sin^2 \theta} - 1\right) n, \tag{24}$$

substituting this value of $D_t \varphi$ in the first of equations (20) we get, by putting R for r,

$$g D_\theta h = R^2 n^2 \sin \theta \cos \theta \left(\frac{4}{9 \sin^4 \theta} - 1\right) - R^2 D_t^2 \theta - g h D_\theta \log \alpha. \tag{25}$$

Since the last term of this equation is a function of h, the height of the strata, the equation can only be satisfied by counter currents of the strata between the equator and the poles; and as we have seen that the figure of the fluid, and its density at the same place, are constant, in order to satisfy the condition of continuity, these currents must be such as to satisfy, for every vertical column of the fluid, the following equation,

$$\int_m D_t \theta' = 0. \tag{26}$$

In order to satisfy this condition, h, which is a general integral, must have a negative constant added to it. Hence at a certain altitude the last term of (20) vanishes, and the fluid there has no motion towards or from the equator.

If the density increases towards the poles, this term is positive for the lower strata, but negative for the upper ones, and hence the motion is toward the equator below, and from it above. If the density decreases towards the poles, the motions are the reverse.

If there were no resistances of any kind, the motions would be continually accelerated so long as the density is different between the equator and the poles; but where there are slight resistances, the motions are only accelerated until the resistances become equal to the accelerating force.

16. Since the last two terms of (25) have a very little effect upon the value of h, and consequently upon the figure of the fluid,

in comparison with the remaining term of that member, unless the difference of density, and the motion of the fluid between the equator and the poles, are very great, we shall neglect them, and determine the figure depending upon the remaining term arising from the earth's rotation. Equation (25) gives by integration in this case, since $D_\phi h$ does not differ sensibly from $D_\theta h$,

$$2gh = -R^2 n^2 \left(\frac{4}{9\sin^2\theta} + \sin^2\theta\right) + C.$$

If h' be put for the value of h at the equator, putting $\sin\theta = 1$, we get

$$2gh' = -\tfrac{13}{9} R^2 n^2 + C.$$

Hence,

$$h = h' + \frac{R^2 n^2}{2g}\left(\tfrac{13}{9} - \frac{4}{9\sin^2\theta} - \sin^2\theta\right). \tag{27}$$

Since one of the terms in this value of h has $\sin\theta$ in the denominator, whatever be the value of h at the equator, it must become 0 towards the poles, and the surface of the fluid meet the surface of the earth; and this must be the case, however large the terms which have been neglected. Hence *the fluid, however deep it may be at the equator, cannot exist near the poles.*

17. If θ_0 be the value of θ where $h = 0$, the last equation gives

$$\sin^4\theta_0 - \left(\frac{2g}{R^2 n^2} h' + \tfrac{13}{9}\right)\sin^2\theta_0 = -\tfrac{4}{9}, \tag{28}$$

which determines θ_0 when h' is given.

If we put θ_1 for the value of θ where h is a maximum, equation (25), putting $D_\phi h = 0$, and neglecting the last two terms, gives

$$\sin^2\theta_1 = \tfrac{2}{3}, \tag{29}$$

This gives $\theta_1 = 55°$ nearly, answering to the parallel of 35°, where h is a maximum.

If, therefore, we assume h', equation (27) gives the figure which the fluid assumes, *which must be somewhat as represented in the external*

part of Fig. (1), *the surface of the fluid being slightly depressed at the equator, having its maximum height about the parallel of* 35°, *and meeting the surface of the earth towards the poles.*

18. In the applications of the preceding equations we must put

$$R = 3956 \text{ miles} = 20887680 \text{ feet},$$

$$n = \frac{2\pi}{(23 \times 60 + 56)\,60} = .000072924$$

$$g = 32.2 \text{ feet}.$$

Hence $Rn = 1523.2$ feet, and $\frac{R^2 n^2}{2g} = 36017$ feet.

With these values, if we assume $h' = 5$ miles, (28) gives $\theta_0 = 28°\ 30'$ for the polar distance of the parallel where the surface of the fluid, or the stratum of equal pressure, meets the surface of the earth.

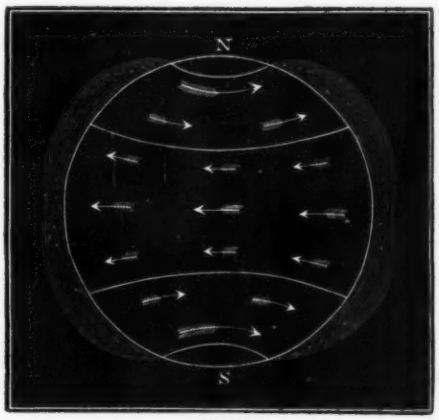

Fig. 1.

If in (27) we substitute for $\sin\theta$ its special value in (29), we obtain $h - h' = 4002$ feet for the excess of the height of the fluid at its maximum, above its height at the equator; which is a constant independent of the amount or depth of the fluid.

19. If we put $D_t\varphi = 0$ in (24), it gives

$$\sin^2\theta = \sin^2\theta_1 = \tfrac{2}{3}. \tag{30}$$

Hence, *the latitude of no motion of the fluid east or west, is the latitude of the maximum of* h.

20. If $\sin^2\theta < \frac{2}{3}$ in (24), $D_t\varphi$ is positive, but if $\sin^2\theta > \frac{2}{3}$, it is negative. Hence, *between the parallels of* 35° *and the poles, the motion of the fluid is eastward, but between those parallels and the equator it is toward the west.*

21. The lineal velocity of the fluid east or west relative to the earth's surface is $R \sin \theta \, D_t \varphi$. Representing it by v'', equation (24) gives

$$v'' = Rn\left(\frac{2}{3 \sin \theta} - \sin \theta\right). \tag{31}$$

Putting $\sin \theta = 1$, this equation gives $u'' = -\frac{1}{3} Rn = 508$ feet for the velocity of the fluid westward per second at the equator. Towards the poles it is evident, from an inspection of the preceding equation, that the velocity must become very great.

The east and west motions of the fluid, as well as its figure, are represented by Fig. (1), the different lengths of arrows representing, in some measure, the different velocities of the fluid.

22. The whole lineal velocity of the fluid east, arising from both the earth's rotation and the velocity of the fluid relative to the earth's surface, is $R \sin \theta \, (n + D_t \varphi)$. Representing this by v''', equation (24) gives

$$v''' = \frac{2Rn}{3 \sin \theta}. \tag{32}$$

Hence this velocity is inversely as the distance from the axis of rotation, which is a necessary consequence of the preservation of areas, as shown in § 13. For as the fluid in moving from the equator towards the poles approaches the axis of rotation, it must have its velocity increased, and in receding from the axis it must be decreased, just as a planet is accelerated in its perihelion but retarded in its aphelion. The reasoning, therefore, of those who, in attempting to explain the trade winds, assume that the fluid, in moving towards or from the equator, has a tendency to retain the same lineal velocity, is erroneous.

23. If, instead of a state of rest relative to the earth's surface, we suppose that the fluid has an initial angular velocity, we must put $n + v'$ instead of n in the preceding equations.

ON PRACTICAL GEOMETRICAL METHODS OF LOCI.

BY D. H. MAHAN,
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EDITOR OF THE MATHEMATICAL MONTHLY:

SIR, — Permit me to call the attention of your younger mathematical readers to the subject of the practical geometrical methods of *loci*, as affording both an interesting and profitable exercise in the use of mathematical instruments, particularly for those who are pursuing scientific studies with the view of becoming civil engineers or architects. I have selected two cases out of the many that have fallen under my notice in their applications to practice.

Prob. To inscribe within a given rectangle another, one side of which is given.

Let $A\,B$ be the given rectangle; $a\,b$ the one to be found, the side $a\,c$ of which is given.

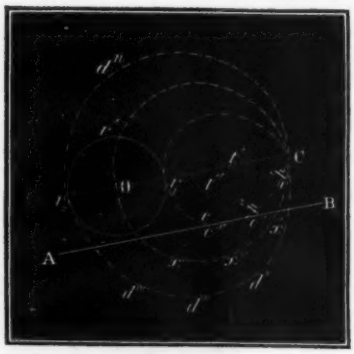
An examination of the Fig. will show that the diagonals of the two rectangles pass through the same point O. From O then, with any assumed radius $O\,a'$ describe an arc, and from a' set off the chord $a'\,c' = a\,c$. If the point c' falls without $B\,C$, take any radius $O\,a'' < O\,a'$ and repeat the same construction, thus determining the point c'' within $B\,C$. Having, in this way, found as many points c', c'' &c., as desired, on either side of $B\,C$ draw through them the line $c'\,c\,c''$, &c. This line will be the *locus* of the required conditions, and the point c, where it intersects $B\,C$, will be one angle of the inscribed rectangle to be found, from which the others are readily determined by the inverse order of the construction.

Remark. This problem finds an application in HOWE'S Truss for

bridges, &c., in determining the position of the oak or iron steps, or blocks against which the diagonal braces abut.

Prob. Having given a circle, a right line and a point without the circle, to find the tangents to the circle to which if perpendiculars be drawn from the given point, the points of intersection of these lines will be found on the given line.

Let O be the centre of the given circle; $O r'$ its radius; C the given point, and $A B$ the given right line. Draw a series of tangents as t', t'', &c., to the circle, and from C demit upon them the perpendiculars p', p'', &c. The points of intersection of these lines will give a curve $c\,c''$, &c., which is the *locus* of the conditions; and where it intersects the line $A B$ as at c and c'' will be the required points.



Remarks. 1st. In examining the data of the Prob. it will be seen that by joining the point C with O the centre of the circle, and on this line, as a diameter describing a circle, and from C drawing any number of chords $C x'$, $C x''$, &c., and on these setting off the distances $x' c'$, $x'' c''$, &c., each equal to $O r'$, the points c', c'', &c., will be the points of the *locus* as before. Moreover, the conditions will still be satisfied by prolonging the chords and setting off upon them the distances $x' d'$, $x'' d''$, &c.; this will give a branch of the curve within which the circle and the point C will be found.

2d. An examination of the preceding constructions will show several of the remarkable properties of this curve, which, *in connection with the circle*, presents two symmetrical *gourd-shaped* branches; $r_2 r' c''$ r_1 $C d''$ being one. The curve will be tangent to the circle at r_1 and r_2, where the line $C O$ cuts the circle; it will pass through the point

C, forming there a cusp or knot. The equation of the curve will evidently be of the 4th degree, with equal pairs of roots for the ordinates referred to an axis perpendicular to $O\,C$, at r_2; and of unequal roots for the abscissas referred to $r_2 C$ as an axis.

3. The problem may evidently be enunciated thus: Having a given chord in a circle, to draw, from a given point, another chord, such that the portion of it intercepted between the given chord and its arc shall be equal to a given line. This problem, if I recall accurately an investigation of it made by me some years back, reduces, for its solution, to the celebrated puzzle of the tri-section of an angle.

This example is also from a practical application; but is given only for the purpose of illustrating the utility of such simple mechanical means for solving problems, otherwise demanding, for their investigation, a resort to analytical geometry which is not at the disposal of many who might resort to this method.

In connection with this subject, I would call the attention of your young readers to the admirable applications of it by my old friend and professor years ago, *General* PONCELET, *Member of the French Institute;* some one of whom would do the scientific civil engineers of our country a great service by translating his *Memoirs on the Stability of Retaining Walls and Arches*, in which he has used these methods in testing two of the most difficult applications of the higher analysis.

To instructors I will permit myself to add, that, as an introduction to practical applications of geometrical constructions, exercises of this character at the black-board, with a scale, dividers, and a triangle, will be found of the greatest service to the young engineer.

Mathematical Monthly Notices.

First Lessons in Geometry. By THOMAS HILL. *Facts Before Reasoning.* Boston: Hickling, Swan and Brown.

OUR readers already know Mr. Hill's views, as set forth in his article "On the order of Mathematical studies," in relation to the early introduction of the study of Geometry, and its value as a mental discipline; first, by means of forms which address the mind through the senses; second, by means of a simple but logical statement of the coördinated series of truths of Geometry, to educate the powers of conception; and lastly, the rigorous demonstration and application of these truths, to strengthen and develop the reasoning and inventive powers. This little book is intended to aid the child in the second stage of its progress in the study of Geometry; and, so far as we know, this is the only one that does not include reasoning and a knowledge of the symbols and operations of arithmetic, which the child at this age is not supposed to have acquired. This idea of putting Geometry before Arithmetic is not new, for Mr. Hill himself has diligently preached the doctrine for the last twelve or fourteen years; but while others may have implied it, we believe he was the first to give it prominence and vitality by putting it into practice. Nor has Mr. Hill confined himself to the simple elements of Geometry, for the "First Lessons" contain a clear and simple statement of the characteristic properties of the Circle, Ellipse, Parabola, Hyperbola, Catenary, the Cycloidal curves, and Caustics, with their Involutes and Evolutes. This book is used in all the schools of Waltham with marked success; but this might be attributed to the energy and enthusiasm which Mr. Hill has infused into these schools by his personal influence, were it not a fact that all thorough teachers, in many schools in different parts of the country who have given the book and the idea which it represents a fair trial, have been equally successful. But on the other hand we feel sure that many teachers would as signally fail; and simply for want of the proper knowledge of, and interest in, the subject. We believe that if this idea is ever thoroughly and generally incorporated into our system of elementary instruction, it will be mainly through the influence of our Normal Schools; the only institutions in which the theory and practice of teaching are studied in connection, and by those who will, in the discharge of the duties of their profession, practise what they believe to be true in theory, whatever may be their natural tastes or preconceived opinions.

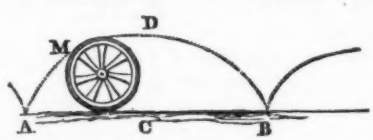
It is generally admitted that one of the greatest, if not the greatest evil at present existing in our primary schools, is the want of proper amusement of an instructive character for small children, while learning the alphabet, and indeed until they have so far advanced as to begin to study for themselves. During this period of unprofitable restlessness and weariness, just the time when pleasant impressions of the school-room, of books, and of study should be received, children often acquire a distaste for these things which they never outgrow; and we believe that hardly another subject can be named so well adapted to amuse and instruct as Geometry taught by means of sets of blocks, and drawing on the slate and blackboard.

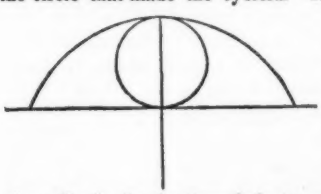
And such a course would be preparatory to the study of the "First Lessons," in connection with which the blocks and drawing would still be used by way of illustrating and simplifying the definitions and descriptions.

The study of Geometry is generally acknowledged as the best means of cultivating the habit of mental attention and concentration; and the desirableness of early disciplining these faculties has led to the wide introduction of the study in all but the lowest grades of schools. Besides, many teachers are agreed that Geometry should precede Algebra, and we find that in many institutions the course of study is arranged accordingly. Mr. Hill proposes to carry the idea still further by making the study of Geometry precede Arithmetic, and to this end has prepared the little work before us. He says "geometrical facts and conceptions are easier to a child than those of arithmetic, but arithmetical reasoning is easier than geometrical." This suggests the order of these studies as a discipline of the reasoning powers. Arithmetical reasoning first, and geometrical afterwards.

We most sincerely commend this little volume and the whole subject to the attention of teachers and all in any way interested in the subject of education, and add the following chapters to show how clear and vivid an idea of even the higher curves can be obtained by description alone, and how decidedly this habit of conceiving of a curve in kind must assist the student afterwards to apply his analysis and compute it in magnitude.

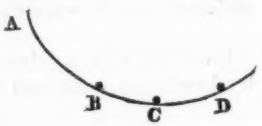
CHAPTER XXVII. *About a Wheel Rolling.* — 1. When a wagon is going upon a straight and level road, look at the head of a spike in the tire of one of the wheels, and you will see that it moves in beautiful curves, making a row of arches that is called a cycloid. 2. That is to say, a cycloid is the path of a point in the circumference of a circle rolling on a straight line. You can draw part of a cycloid by putting the point of your pencil into a little notch in the edge of a spool, and tying it fast, so that the point of the pencil shall be kept just at the edge of the spool; and then rolling the spool carefully and slowly against the inside of the frame of the slate. 3. You will see, I think, that each arch in the cycloid must be just as high from C to D as the diameter of the circle that makes it; and just as wide at the bottom, from A to B, as the whole circumference of the circle. 4. But you will have to study Geometry a good while, before you can prove the other interesting things which I am going to tell you. You can easily understand what I am going to tell you; but you cannot understand how I know it, as you can what I told you in the last section. 5. The length of the curve, A D B, in each arch of a cycloid, is just four times the height of the arch; that is, four times the diameter of the circle that made the cycloid. 6. The whole space that is enclosed between the arch of the cycloid and the straight line on which it stands, is just three times as large as the circle that made the cycloid. So, when a circle is inscribed between the arch and the line, the curious three-cornered figures on each side of the circle are each exactly as large as the circle itself. 7. Now, if you have studied Arithmetic, you will understand that, if a wheel is three feet in diameter, the head of a spike in the tire travels just twelve feet from where it leaves the ground until it touches the ground again. The spots where it touches the earth will be nine feet and three sevenths of a foot apart. And the space between its path and the ground will be three times eleven fourteenths of nine square feet; that is, twenty-one square feet and three fourteenths of a square foot.



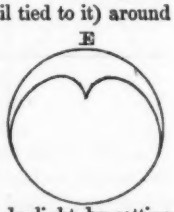


CHAPTER XXVIII. *More about a Rolling Wheel.* — 1. The head of a spike in the tire of a

rolling wheel is moving, at each instant, at right angles to a line joining it to the bottom of the wheel. 2. That is to say, if a straight line is drawn from the bottom of the rolling wheel to the head of the spike, and if a tangent to the cycloid is drawn through the head of the spike, this straight line will be at right angles to this tangent. 3. And this straight line is exactly half of the radius of curvature of that point in the cycloid. So that at the top of an arch of the cycloid the radius of curvature will be twice the diameter of the circle, and as you go down the arch the radius of curvature will be shorter and shorter, until just at the foot of the arch the radius of curvature will be of no length at all. 4. The evolute of a cycloid is a cycloid of exactly the same size. That is to say, if we should fasten a string in the point between two arches of a cycloid, just long enough to wrap on the curve up to the middle of the arches, its end as it is wrapped and unwrapped, would move in a cycloid exactly like that to which it was fastened. 5. If a cycloid be turned upside down, and we fancy the inside of it to be very exceedingly slippery, then there are two curious things about it. If I want to slide any thing from A down to B, there is no curve, nor straight line, down which a thing would slide so quickly as down the cycloid. If a hill was hollowed out in that shape, sleds would run down it faster than they could down any other shaped hill of the same height and the same breadth at the bottom. 6. The second curious thing about sliding on the inside of a cycloid is, that it takes always exactly the same time to slide to the bottom, however high up or low down you start. If A, in the last figure, is the top of such a hill, and C the lowest point, it will take a sled exactly as long to go from B to C, as to go from A to C. But this, you must remember, is only when we imagine the hill and the runners of the sleds to be, both of them, perfectly slippery; so that there shall be no rubbing. In that case, if the road from A to C was two miles long, it would only take a sled twenty-eight seconds to come down the whole length. And, if it starts from any other place on the road, say from B, it will still take twenty-eight seconds to get to C. 7. If the road from A to C is half a mile long, a sled will come down in fourteen seconds. 8. If a board is sawed out in the form of a cycloid, and a little gutter made on the inside of the curve, you can try this by holding two marbles, say one at A and the other at D, and letting go of them at the same instant. They will meet exactly at C, one coming the whole way A C, while the other is coming the short distance D C.

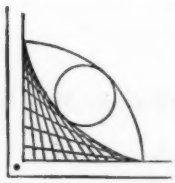


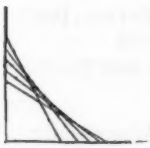
CHAPTER XXIX. *Wheels Rolling Round a Wheel.* — 1. When one circle rolls around another, instead of rolling on a straight line, any point in the circumference of the rolling circle travels in a curve called an epicycloid. You can draw an epicycloid by rolling carefully the spool (with the pencil tied to it) around some round thing held still on your slate. 2. Set a lamp on a table in one corner of the room, and, in the farthest corner of the room, on a table of nearly the same height, set a bright tin cup, or a glass tumbler, nearly full of milk. On the surface of the milk you will see a bright curve shaped like the inner line in this figure. It is an epicycloid; such as would be made by a circle of one quarter the diameter of the cup rolling on a circle half the size of the cup. You can make it by daylight, by setting the cup of milk in the sunshine, early in the morning or late in the afternoon. 3. Epicycloids



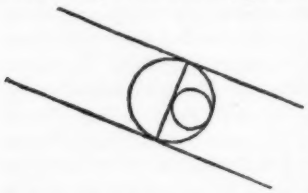
will be of different shapes, according to the proportion which the two circles bear to each other. The smaller the rolling circle is in proportion to the other, the more nearly will an arch of the epicycloid be like an arch of the cycloid. 4. The evolute of an epicycloid is a smaller epicycloid of the same shape; and the evolute of that evolute must be a still smaller epicycloid. So that we may fancy epicycloids packed one within another like pill-boxes. 5. The epicycloid of section second is sometimes called by children the cow's foot in a cup of milk. The figure in the margin represents this epicycloid with its nest of evolutes packed one within the other. If a string is fastened at the point where the arches of the epicycloid come together, and is just long enough to wrap round to the middle of the arch, then, as it unwraps, the end will move in a larger epicycloid of exactly the same shape. 6. When the circles are of the same size, the epicycloid will have but one arch. The ends of the arch will come together at the same point. The figure in the margin will show the shape of this epicycloid and its evolutes.

CHAPTER XXX. — *Of a Wheel rolling on the inside of a Hoop.* — 1. When a circle rolls on the inside of another circle, instead of on the outside, the curve is called a hypocycloid. 2. Suppose your slate-pencil were straight, so that it would lie flat on the slate, and make a mark as broad as the pencil is long. Then suppose you were to put your pencil across one corner of your slate like a hypotenuse, and slide first one end up to the corner, and then the other, keeping both ends all the time touching the slate frame. You would make a white mark in the corner, of a curved three-cornered shape, like this figure. The curve inside would be a hypocycloid. 3. If you take the corner of your slate for a centre, and the length of your pencil for a radius, and draw a quarter of a circle, as I have done in the last figure; if you then roll on the inside of this arc a circle whose diameter is one half the length of the pencil, it will make the same hypocycloid. I have also drawn this circle in the figure. 4. You can draw a hypocycloid by rolling the spool and the pencil on the inside of any little hoop held firmly on the slate. The rim of the cover of a large wooden pill-box will make a nice little hoop for this purpose. 5. The evolute of a hypocycloid is a larger hypocycloid of the same shape on the outside of it; and the hypocycloid itself may be fancied as the evolute of a smaller hypocycloid within it; so that hypocycloids, like epicycloids, are packed one within the other, like nests of tubs or boxes. 6. The hypocycloid that is made when the diameter of the rolling circle is one quarter of the diameter of the circle that it rolls in, can be made by sliding the hypotenuse backwards and forwards on the legs of a right triangle. The hypotenuse must be kept of the same length, and it will always be a tangent to the hypocycloid. 7. This hypocycloid may be called a hypocycloid of four arches; because, as you may see in the figure, both it and its evolutes have each four arches. 8. If the diameter of the spool is nearly half that of the hoop, the pencil will move across the hoop in a very flat curve, almost like a diameter of the hoop; and the evolute at the ends of the curve will be almost like two parallel straight lines at right angles to the end of the diameter; so that the string unwrapping from the evolute will be very long. When the diameter of the spool is exactly half that of the hoop, the hypocycloid is a straight line; and the evolute of it, if you can fancy that





there is any evolute, is two parallel straight lines at right angles to its ends. 9. If the diameter of the spool is more than half that of the hoop, it will make a hypocycloid like that made by a smaller spool. If you have two spools, one of them as much wider than the radius of the hoop as the other is smaller, so that the hoop will just let the two spools stand in it side by side, then one spool will make exactly the same hypocycloid as the other. 10. These two spools cannot, of course, be both rolling in the hoop at the same time; but we can easily imagine two circles of the same size as the spools rolling in a circle as large as the hoop. Start the circles from the position in which I have drawn them to rolling in opposite directions, and if you roll the little circle faster than the large one, so as to make them get round the hoop in the same time, the points in the two circles which are now touching will keep together all the time, making the same hypocycloid.



Editorial Items.

The following gentlemen have sent us Solutions of the Prize Problems in the December No. of the Monthly.

Arthur Wilkinson, Junior Class, Harvard College, answered all the questions but I.

George B. Hicks, Student, Cleveland, Ohio, answered all the questions.

David Trowbridge, Student, Perry City, Schuyler Co., N. Y., answered all the questions.

James M. Ingalls, Delton, Sauk Co., Wisconsin, answered questions IV. and V.

W. H. Stirling, Student, Baltimore, Md., answered problem I.; also sent solutions of problem II. in October No., problem III. in November No., and problem I. in January No.

Asher B. Evans, Junior Class, Madison Univ., Hamilton, N. Y., answered all the questions.

G. W. Hill, Senior Class, Rutger's College, New Brunswick, N. J., answered all the questions.

A Student in Baltimore College answered all the questions. (Richard Cotter, Prof.)

E. P. Austin, Student in the Univ. of Michigan, answered all but I. and III.

We shall publish the report of the Judges, with the Prize Solutions, in the next Number.

Contents *of the Nouvelles Annales de Mathématiques*, January, 1859.

Fundamental Formulas of Spherical Analysis; by M. Vannson. Upon Interpolation; by M. Eugène Rouché. Application of the New Analysis (determinants) to Surfaces of the Second Order; by M. Painvin. Note by M. Lebesgue. Questions Proposed; by M. Vannson. Focal Properties of Surfaces of the Second Order, after M. Hellermann; by M. Dewulf.

Bulletin *de Bibliographie, d'Histoire et de Biographie Mathématiques.*

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